# EXPERIMENTAL INVESTIGATION OF THE INFLUENCE OF THE OUTFLOW PARAMETERS OF TWO-DUCT, NONISOTHERMAL JETS ON THE LEVEL OF SOUND PRESSURE AND THE FREQUENCY SPECTRUM OF EMITTED NOISE

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Generalizing relations are derived for the acoustic characteristics of a subsonic, nonisothermal, two-duct jet as functions of its geometrical and gas-dynamic parameters, enabling one to determine the sound pressure level and its frequency spectrum in the far acoustic field.

1. In [1-4], devoted to the acoustic characteristics of two-duct jets, various empirical relations have been proposed that generalize the results of measurements and permit and approximate calculation of the acoustic power. The purpose of such research, as a rule, is to obtain data from which the acoustic characteristics of the jets from tw-duct aircraft engines may be determined. To determine aircraft noise, however, one must have data on the directivity of the sound emission and its frequency spectrum in addition to data on the acoustic power level of the jet.

In [4] we showed that the concept of an "equivalent jet" enables one to generalize with satisfactory accuracy the data of measurements of acoustic power. In the present paper we continue the research of [4], using the same experimental conditions and measurement procedure, to determine the sound pressure levels and noise spectra in the far acoustic field of a jet with stepped profiles of gas velocity and temperature in its initial cross section. On the basis of the relations obtained in [4] for the equivalent density $\rho_e$ and velocity $V_e$ as functions of the parameters of a two-duct jet, we determine the proportionality factor, enabling us to calculate the acoustic power using Lighthill's equation, and we obtain the dependence of the directivity factor of sound emission on the parameter $V_eV_1/a_aV_2$ and the characteristic frequency, determined from the parameter

$$\mathrm{Sh}_e = f\,\frac{D_e a_e}{V_e a_a}.$$

2. For the experimental models we used four two-duct nozzles, the flow sections of which were profiled using Vitoshinskii's equation. The ratios of the area $F_2$ of the outer duct to the area $F_1$ of the inner duct were 0.5, 1, 2, and 3. The experimental setup permitted autonomous variation of the total pressure at the nozzle entrance from $1.2\cdot10^5$ to $1.9\cdot10^5$ Pa and the gas temperature from 273 to 800 K in both the inner and outer nozzle ducts. The sound pressure levels were measured with an electroacoustic instrument, the microphone of which was moved with a positioner along a circle of radius 6 m from the nozzle cut in a horizontal plane passing through the nozzle axis.

3. To generalize the data of acoustical power measurements in [4], the investigated jet was set in correspondence with a certain equivalent jet with the same flow rate, impulse, and initial cross section area. The gas density $\rho_e$ and outflow velocity $V_e$ for the equivalent jet are determined from the conditions of equality of the flow rate, momentum, and static pressure in the investigated and equivalent jets.

In this case, $\rho_e$ and $V_e$ are calculated from the equations

$$\rho_e = \frac{p_a}{R_g\,\tau(\lambda_1)}\;\frac{\left(\dfrac{\lambda_1}{\sqrt{\overline{T}_1}} + \dfrac{\lambda_2}{\sqrt{\overline{T}_2}}\,\dfrac{\tau(\lambda_1)}{\tau(\lambda_2)}\,\overline{F}\right)^2}{\left(\lambda_1^2 + \lambda_2^2\,\dfrac{\tau(\lambda_1)}{\tau(\lambda_2)}\,\overline{F}\right)(1+\overline{F})}, \tag{1}$$

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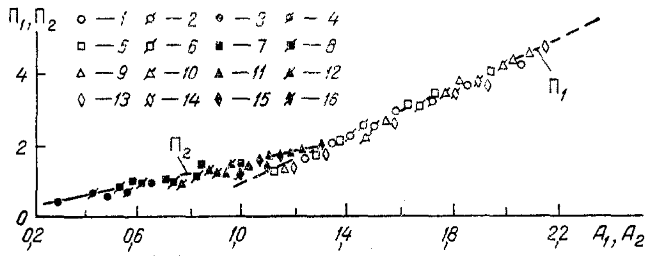


Fig. 1. Comparison of experimental data with approximations of the parameters $\Pi_1$ and $\Pi_2$. 1-4) $\bar{F} = 0.5$: 1) $T_1/T_2 = 573/300$; 2) 773/300; 3) 300/573; 4) 300-773; 5-8) $\bar{F} = 1$; 9-12) $\bar{F} = 2$; 13-16) $\bar{F} = 3$ for the same ratios $T_1/T_2$ as for $\bar{F} = 0.5$.

$$V_e = \sqrt{\frac{2\varkappa}{\varkappa+1} R_g} \frac{\lambda_1^2 + \lambda_2^2 \frac{\tau(\lambda_1)}{\tau(\lambda_2)} \bar{F}}{\frac{\lambda_1}{\sqrt{T_1}} + \frac{\lambda_2}{\sqrt{T_2}} \frac{\tau(\lambda_1)}{\tau(\lambda_2)} \bar{F}}. \tag{2}$$

According to Lighthill's theory, the acoustic power of a submerged gas jet is

$$W_0 = K a_a^{-5} \rho_0 V_0^8 F_0, \tag{3}$$

where K is an experimental coefficient that depends on the geometrical parameters of the nozzle ($K = 0.8 \cdot 10^{-4}$ for test models of a one-duct jet). An analysis of experimental data obtained in the present work and in [4], in order to find a dimensionless proportionality factor with which the acoustic power of a two-duct nonisothermal jet can be calculated using Eqs. (1), (2) and (3), showed that the quantity $\Pi = \varphi\ (J_1/J_\Sigma;\ J_2/J_\Sigma;\ T_1;\ T_2;\ \bar{F})$, where $J_\Sigma$, $J_1$, and $J_2$ are the total impulse and the impulses of the streams escaping from the inner and outer ducts of the nozzle, respectively, may be such a parameter. In this case, $\pi$ is a correction to Lighthill's coefficient K, and the acoustic power of a two-duct nonisothermal jet may be calculated from the equation

$$W = \Pi K a_a^{-5} \rho_e V_e^8 (F_1 + F_2). \tag{4}$$

It has been established from experimental data [4] that the parameter $\Pi = \Pi_1$ for an ordinary velocity profile ($V_1 > V_2$) differs from the parameter $\Pi = \Pi_2$ for an "inverted" velocity profile ($V_2 > V_1$), with

$$\Pi_1 = \varphi_1 \left[\frac{J_1}{J_\Sigma}(1+\bar{F})\sqrt{\frac{T_1}{T_2}}\right] \text{and } \Pi_2 = \varphi_2 \left[\frac{J_2}{J_\Sigma}\sqrt{\frac{T_2}{T_1}}\right].$$

The form of the function $\varphi_1$ and $\varphi_2$ can be determined from the experimental data of [4] with sufficient accuracy for engineering calculations by approximating the dependence of $\Pi_1$ and $\Pi_2$ on the determining parameters using the relations

$$\Pi_1 = 3.25 A_1 - 2.25; \tag{5}$$

$$\Pi_2 = 1.5 A_2 + 0.05, \tag{6}$$

where

$$A_1 = \frac{(1+\bar{F})\sqrt{\frac{T_1}{T_2}}}{\frac{1+\lambda_2^2}{1+\lambda_1^2} \frac{\tau(\lambda_1)}{\tau(\lambda_2)} \bar{F} + 1};$$

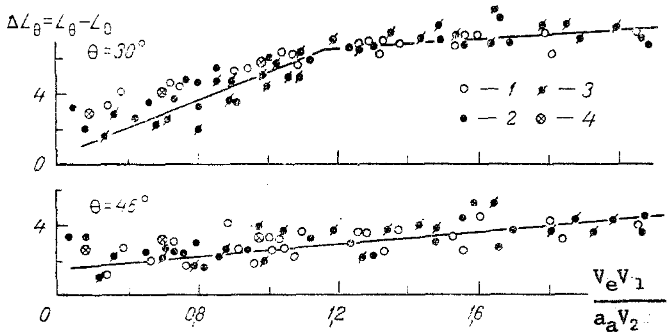


Fig. 2. Directivity factor $\Delta L_\theta$ as a function of the parameter $V_eV_1/a_aV_2$ for θ = 30 and 45°: 1) $\bar{F}$ = 0.5-1; 2) $\bar{F}$ = 2; 3) $\bar{F}$ = 3 for values of $T_1$ and $T_2$ ranging from 300 to 773 K; 4) with $V_1 = V_2$ and $T_1 = T_2$ for $\bar{F}$ = 1. $\Delta L_\theta$, dB.

$$A_2 = \frac{\bar{F}\sqrt{\frac{T_2}{T_1}}}{\frac{1+\lambda_1^2}{1+\lambda_2^2}\frac{\tau(\lambda_2)}{\tau(\lambda_1)} + \bar{F}}.$$

The approximations (5) and (6) are compared with experimental data in Fig. 1. One may see that their accuracy is of order 1 dB. The acoustic power emitted by a two-duct nonisothermal jet can thus be calculated from the values of λ, τ(λ), $T_1$, $T_2$, $F_1$, and $\bar{F}$ in the initial cross section of the jets escaping from the inner and outer ducts of the nozzle, using relations (5) and (6), from the following equations: for $V_1 \geq V_2$

$$W = \frac{K\Pi_1\, 12\cdot 10^5 P_a F_1}{\tau(\lambda_1) T_a^{2,5}} \frac{\left(\lambda_1^2 + \lambda_2^2 \frac{\tau(\lambda_1)}{\tau(\lambda_2)}\bar{F}\right)^7}{\left(\frac{\lambda_1}{\sqrt{T_1}} + \frac{\lambda_2}{\sqrt{T_2}}\frac{\tau(\lambda_1)}{\tau(\lambda_2)}\bar{F}\right)^6}, \tag{7}$$

and for $V_2 > V_1$

$$W = \frac{K\Pi_2\, 12\cdot 10^5 P_a F_1}{\tau(\lambda_1) T_a^{2,5}} \frac{\left(\lambda_1^2 + \lambda_2^2 \frac{\tau(\lambda_1)}{\tau(\lambda_2)}\bar{F}\right)^7}{\left(\frac{\lambda_1}{\sqrt{T_1}} + \frac{\lambda_2}{\sqrt{T_2}}\frac{\tau(\lambda_1)}{\tau(\lambda_2)}\bar{F}\right)^6}. \tag{8}$$

4. To calculate the sound pressure level at a certain point of the acoustic field for a known acoustic power W, one must know the directivity factor, which has come to be represented in the form of the difference between the sound pressure level $L_\theta$ in the direction toward the control point at the angle θ relative to the jet axis and the sound pressure level $L_0$ at the same point that is emitted by a source of the same acoustic power but having a circular directivity characteristic.

It is well known that the formation of the directivity of acoustic emission from a gas jet is affected by the "intrinsic" noise, the "shifted" noise, convection, and refraction. The formation of the directivity of acoustic emission from a submerged jet has been analyzed in detail in [1]. There it is also shown that the difference in levels $\Delta L_\theta = L_\theta - L_0$ can be represented in the form of a generalized function $\Delta L_\theta(V/a_a)$ for different directivity angles θ.

An analysis of the large amount of experimental data obtained in the present work enabled us to establish that for nonisothermal two-duct jets with a stepped nonuniformity in the velocity profile in the initial cross section of the jet, $\Delta L_\theta(V_eV_1/a_aV_2)$ can also be represented in the form of a generalized relation for different directivity angles θ. It must be noted that for $V_1 = V_2 = V_0$ and $T_1 = T_2 = T_0$, this relation should coincide with the relation $\Delta L_\theta(V_0/a_a)$ for a one-duct jet. As we shall show below, such agreement occurs for $V_eV_1/a_aV_2 < 1$, since for higher values of this parameter a one-duct jet will have a supersonic

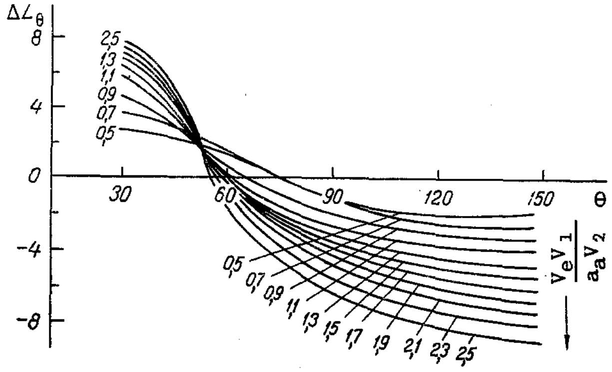


Fig. 3. Directivity factor $\Delta L_\theta$ as a function of the angle θ for different values of the parameter $V_eV_1/a_aV_2$. θ, deg.

outflow regime. In Fig. 2 we show values of the directivity factor $\Delta L_\theta$ for the ranges P = $1.2\cdot10^5$–$1.9\cdot10^5$ Pa and T = 273-800 K in both the inner and the outer ducts of two-duct nozzles with area ratios $\bar{F}$ = 0.5, 1, 2, and 3 as functions of the parameter $V_eV_1/a_aV_2$. From this figure one may see that, for a fixed value of the angle θ, this parameter can be used to generalize measurements of the directivity factor $\Delta L_\theta$ for the sound pressure level emitted by a nonisothermal two-duct jet. In Fig. 2 we also show $\Delta L_\theta$ for a one-duct jet, i.e., with $V_1 = 2_2$ and $T_1 = T_2$. One may see that the function $\Delta L_\theta(V_eV_1/a_aV_2)$ for this case coincides with the function $\Delta L_\theta(V_0/a_a)$.

The sound pressure level L at points of the acoustic field located at a distance r from the nozzle cut at an angle θ to the jet axis may be calculated from the equation

$$L_\theta = 10\lg\frac{W}{W_0} - 10\lg 4\pi r^2 + \Delta L_\theta, \tag{9}$$

where W is the acoustic power calculated from Eq. (7) or (8); $W_0 = 10^{-12}$ W is the threshold value of the acoustic power; $\Delta L_\theta$ is the directivity factor.

Values of $\Delta L_\theta$ (obtained from experimental data like those in Fig. 2) are presented in Fig. 3 in the form of functions of the angle θ for different velocity ratios. These functions were obtained by replotting the generalized relations $\Delta L_\theta(V_eV_1/a_aV_2)$ for different fixed values,

$$\frac{V_eV_1}{a_aV_2} = \frac{0.9}{\sqrt{T_a}}\frac{\lambda_1}{\lambda_2}\sqrt{\frac{T_1}{T_2}}\,\frac{\lambda_1^2 + \lambda_2^2\frac{\tau(\lambda_1)}{\tau(\lambda_2)}\bar{F}}{\frac{\lambda_1}{\sqrt{T_1}} + \frac{\lambda_2}{\sqrt{T_2}}\frac{\tau(\lambda_1)}{\tau(\lambda_2)}\bar{F}}. \tag{10}$$

Comparing the sound pressure levels L calculated from Eq. (9) with experimental data obtained in special control tests showed that the difference between the measured and calculated L was within the accuracy limits of the acoustical instrument from the Bruel and Kjeer Co. (±1 dB).

5. In [1] it was shown that the spectral characteristics of the noise from a two-duct isothermal jet can be represented in generalized form by the function $\Delta L_i(Sh)$, where $\Delta L_i = L_i - L_\Sigma$, while $Sh_1 = fD/V_1$ is the Strouhal number for an ordinary velocity profile ($V_1 > V_2$) and $Sh_2 = fD'/V_2$ is that for an "inverted" velocity profile ($V_2 > V_1$). Here D is the nozzle diameter for a certain jet that is determined from the condition of equal momentum:

$$\rho_1 V_1^2 D_1^2 + \rho_2 V_2^2 (D_2^2 - D_1^2) = \rho_1 V_1^2 D^2 \text{ for } V_1 > V_2$$
$$\text{and } \rho_1 V_1^2 D_1^2 + \rho_2 V_2^2 (D_2^2 - D_1^2) = \rho_2 V_2^2 (D^1)^2 \text{ for } V_2 > V_1.$$

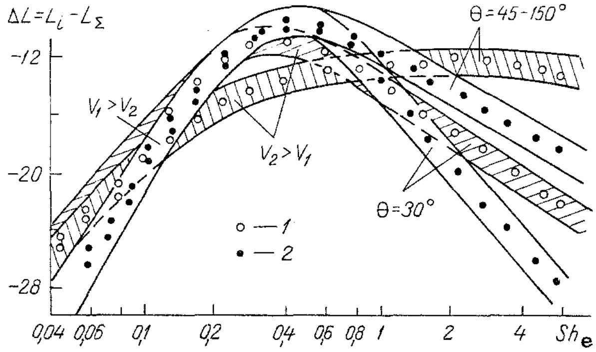


Fig. 4. Spectra of sound pressure levels from a nonisothermal two-duct jet for $V_2 > V_1$ and for $V_1 > V_2$: 1) control tests for $V_2/V_1 = 1.15$; 2) the same for $V_2/V_1 = 0.56$ and $\bar{F} = 1$. ΔL, dB.

An analysis of a large amount of experimental data, including those obtained in the pre present work, showed, however, that the measured results for nonisothermal two-duct jets can be generalized better if the spectral characteristics are represented by the function $\Delta L_i$·($SH_e$), where $Sh_e = fD_e a_e / a_a V_e$ is a certain "equivalent" Strouhal number. In engineering calculations it is convenient to calculate this parameter using gas-dynamic functions:

$$Sh_e = 0.054 f D_e \sqrt{\frac{(1+\bar{F})\,\tau(\lambda_1)}{T_a\left(\lambda_1^2 + \lambda_2^2\,\frac{\tau(\lambda_1)}{\tau(\lambda_2)}\,\bar{F}\right)}}.$$

All the measured values of $\Delta L_i$ for $\bar{F} = 0.5$, 1, 2, and 3 with variation of the pressures $P_1$ and $P_2$ from $1.2\cdot10^5$ to $1.9\cdot10^5$ Pa and of the temperatures $T_1$ and $T_2$ from 280 to 800 K in both the inner and outer nozzle ducts are generalized and represented in Fig. 4 as a function of the Strouhal number $Sh_e$ by bands of width ±2 dB for an ordinary profile ($V_1 > V_2$) and for an "inverted" velocity profile ($V_2 > V_1$). The experimental data (Fig. 4) show that the functions $\Delta L_i(Sh_e)$ for all directions of sound emission are the same except for the direction $\theta = 30°$, for which the sound pressure level was highest. The values of $\Delta L_i$ obtained in the control tests coincided with the generalized functions $\Delta L_i(Sh_e)$, as seen from Fig. 4.

## NOTATION

Gas-dynamic and geometrical parameters in the initial cross section of the jet: P, total pressure, Pa; T, stagnation temperature, K; V, velocity, m/sec; ρ, density, kg/m³; isoentropic index; a, speed of sound, m/sec; $a_{cr}$, critical speed, m/sec; $\lambda = V/a_{cr}$, reduced velocity; $\tau(\lambda) = 1 - \frac{\varkappa-1}{\varkappa+1}\lambda^2$ gas-dynamic function; $R_g$, gas constant, J/(kG·K); F, area, m²; $\bar{F} = F_2/F_1$, area ratio; $D_e = \sqrt{4(F_1 + F_2)/\pi}$, diameter of the equivalent jet, m. Acoustic parameters: W, acoustic power, W; $W_0 = 10^{-12}$ W, threshold value of the acoustic power; f, frequency, Hz; $L_i$, sound pressure level in the i-th frequency band of the one-third-octave filter; $L_\Sigma$, total sound pressure level; $L_\theta$, sound pressure level in the direction of the control point at an angle θ relative to the jet axis; $L_0$, sound pressure level at the control point produced by a source having a circular directivity characteristic. Indices: 0, one-duct jet; 1, inner duct; 2, outer duct; a, ambient medium (the atmosphere); e, equivalent jet.

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# NEAR-WALL TURBULENCE IN THE AXISYMMETRIC FLOW OF WEAK ACQUEOUS POLYMER SOLUTIONS

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The axisymmetric turbulent boundary layer is computed by using a finite-difference method and its fluctuation characteristics are determined on the basis of a generalized mixing-path hypothesis.

1. Investigation of turbulent exchange, including for polymer solution flows, is one of the important problems of applied modern hydromechanics. In addition to experimental investigations ([1, 2], say) it is expedient to produce a computational method that permits finding the fluctuation characteristics of different flows with a definite reliability. Such a method can be based on the generalized mixing-path hypothesis. Thus the possibility is shown in [3] of the possibility of describing the fluctuation characteristics in the near-wall layer of constant stress, pipes, and a flat plate boundary layer.

However, in the majority of cases the longitudinal pressure gradient and the three-dimensional nature of the flow influence the boundary layer development. Both these factors (the second not in the complete volume it is true but only in the ratio of the leakage-spread ratio of the retarded mass of the fluid) hold in an axisymmetric boundary layer whose analysis is comparatively simple because of its formal two-dimensionality. The analysis can here be performed within the framework of the conception of a thick layer by both an integral and finite-difference method [4], of which the latter permits the description of derivatives of the averaged velocity components and their associated tangential stress distributions in boundary layer sections with an accuracy sufficient for a subsequent calculation of the fluctuation characteristics.

The analysis of a thick axisymmetric turbulent boundary layer is performed in this paper for both a Newtonian fluid and for weak polymer solutions with the viscous-nonviscous interaction of the layer and wake with the external potential stream taken into account. Influence of the potential part of the flow on the boundary layer and wake parameters is taken into account in terms of the velocity distribution over their external boundary. To take account of the action of the viscous flow domain on the potential flow the latter is computed on a semi-infinite body formed by the external boundary layer boundary and the wake on which values of the normal velocity component found by means of the boundary layer and wake parameters are given. It is assumed that polymer admixtures influence the turbulent wake parameters only in terms of a change in the boundary layer characteristics at the site of layer and wake juncture, at the root extremity of the body. The action of the polymer admixtures on the potential flow is taken into account in terms of the change in layer and wake characteristics and the location of their external boundaries. The method in [5] is used to compute the wake and the method in [6], approved in [7] for the case of weak polymer solutions, is used to compute the potential domain.

The axisymmetric boundary layer equation in dimensionless form in Crocco variables can be written in the form [8]

$$\frac{\partial\omega}{\partial x} = \lambda_1 \frac{\partial^2(\lambda_3\omega)}{\partial u^2} + \lambda_2 \frac{\partial\omega}{\partial u} + \lambda_4 \quad (1)$$

with boundary conditions for the impenetrable surface

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